

Generic conditions for stable hybrid stars

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We study the stability and maximum mass of hybrid stars, assuming a generic quark matter equation of state with a single first-order phase transition between nuclear and quark matter, and a sharp interface between the quark matter core and nuclear matter mantle in a neutron star. For standard nuclear matter equations of state, we find that the mass-radius relation contains a stable hybrid branch, connected to the nuclear matter star branch, if the energy density discontinuity at the nuclear-quark transition is less than a critical value, which is typically between 60% and 80% of the nuclear matter energy density at the transition.

Extending the quark matter equation of state to higher densities by assuming it has a density-independent speed of sound, we find that, as has been noticed before, there can be a disconnected branch of hybrid stars (sometimes called “third family” of stars). For typical nuclear matter equations of state, this branch exists if the nuclear matter density at the transition is less than a critical value which is about two to four times nuclear saturation density. We calculate the maximum hybrid star mass as a function of the nuclear matter density and the parameters of the quark matter EoS, and find that, for apparently reasonable values of the quark matter parameters, hybrid stars with mass above $2 M_{\odot}$ can exist.

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I. INTRODUCTION

There has been much work on a plausible form for the equation of state (EoS) of nuclear matter at densities above nuclear density, using models of the nuclear force that are constrained by existing scattering data (see, for example, [1–3]). However, we remain almost completely ignorant of the quark matter EoS at the low temperatures and the range of densities that are of relevance to neutron stars. This is because cold quark matter cannot be created in laboratories, and numerical studies using the known strong interaction Lagrangian are stymied by the sign problem (see, for example, [4–7]).

In this work we therefore assume a generic quark matter equation of state, and see in what ways it might be constrained by measurements of the mass and radii of compact stars. We assume that there is a first-order phase transition between nuclear and quark matter, and that the surface tension of the interface is high enough to ensure that the transition occurs at a sharp interface (Maxwell construction) not via a mixed phase (Gibbs construction). This is a possible scenario, given the uncertainties in the value of the surface tension [8, 9]. (For analysis of generic equations of state that continuously interpolate between the phases to model mixing or percolation, see Refs. [10, 11].)

We then answer the following questions:

1. What is a sufficient condition for a completely generic quark matter equation of state to yield a stable branch of hybrid stars?
2. If the quark matter EoS is not completely generic

but the only assumption we make about it is that it has a density-independent speed of sound (like a classical ideal gas [29]), what form does the hybrid star branch take, and how heavy can those stars be?

To address these questions we need to assume a nuclear matter EoS, but for the first question we only need to know some generic features of the quark matter EoS, namely the pressure $p = p_{\text{crit}}$ at which the transition from nuclear matter to quark matter occurs, and the discontinuity in energy density $\Delta\epsilon$ at the transition (Fig. 1). Our answer to the first question will therefore depend on p_{crit} and $\Delta\epsilon$ but it will be model-independent, requiring no additional assumptions about the quark matter equation of state.

To address the second question, we will need the quark matter equation of state over a range of pressures above p_{crit} . In this paper we assume, that the quark matter has a classical-ideal-gas equation of state, for which the speed of sound c_{QM} remains constant as the pressure varies from p_{crit} up to the central pressure of the maximum mass star. The equation of state for dense matter is therefore approximated as (see Fig. 1)

$$\epsilon(p) = \begin{cases} \epsilon_{\text{NM}}(p) & p < p_{\text{crit}} \\ \epsilon_{\text{NM}}(p_{\text{crit}}) + \Delta\epsilon + c_{\text{QM}}^{-2}(p - p_{\text{crit}}) & p > p_{\text{crit}} \end{cases} \quad (1)$$

where $\epsilon_{\text{NM}}(p)$ is the nuclear matter equation of state. In Appendix A we describe the thermodynamically consistent parameterization of the EoS that we used for quark matter. A similar generic parameterization is used in Ref. [12], which also considered the possibility of two

EoS	max mass	radius at $M = 1.4 M_\odot$	L
NL3	$2.77 M_\odot$	14.92 km	118 MeV
HLPS	$2.15 M_\odot$	10.88 km	33 MeV

TABLE I: Properties of the NL3 and HLPS equations of state. L characterizes the density-dependence of the symmetry energy (see text). NL3 is an example of a stiff EoS, HLPS is an example of a softer one at density $n \lesssim 4n_0$.

first-order transitions involving two different phases of quark matter.

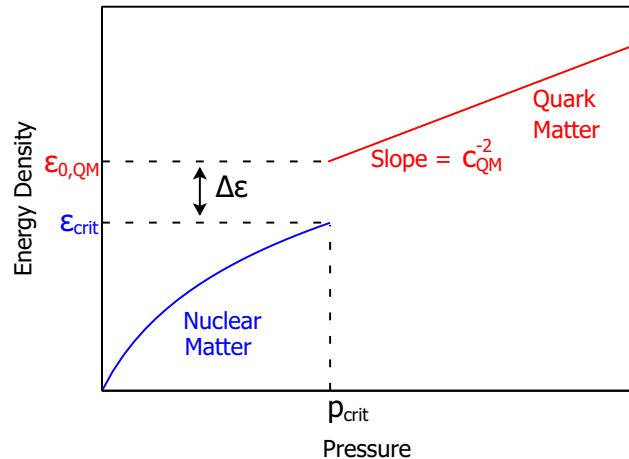


FIG. 1: Equation of state $\varepsilon(p)$ for dense matter. The quark matter EoS is specified by the critical pressure p_{crit} , the energy density discontinuity $\Delta\varepsilon$, and the speed of sound in quark matter c_{QM} (assumed density-independent). Below p_{crit} , the NL3 EoS (curved line) is shown for illustration.

- **Quark Matter EoS.** The assumption that quark matter has a density-independent speed of sound is reasonably consistent with some well-known quark matter equations of state. For some Nambu–Jona-Lasinio models, the EoS fits Eq. (1) almost exactly [12]. In addition, the perturbative quark matter EoS [13] has roughly density-independent c_{QM}^2 , with a value around 0.2 to 0.3, above the transition from nuclear matter (see Fig. 9 of Ref. [14]). In the quartic polynomial parameterization [15], varying the coefficient a_2 between $\pm(150\text{MeV})^2$, and the coefficient a_4 between 0.6 and 1, and keeping n_{crit}/n_0 above 1.5 ($n_0 \equiv 0.16\text{fm}^{-3}$ is the nuclear saturation density), one finds that c_{QM}^2 is always between 0.3 and 0.36.

In this paper we study the maximum hybrid star mass for a range of values of c_{QM}^2 , from $1/3$ (characteristic of very weakly interacting massless quarks) to 1 (the maximum value consistent with causality). We hope that this will give us a reasonable idea of the likely range of outcomes for realistic quark matter in which c_{QM} could vary with density.

- **Nuclear Matter EoS.** Up to nuclear saturation density, the nuclear matter EoS can be experimentally constrained. If one wants to extrapolate it to densities above

n_0 , there are many proposals in the literature. For illustrative purposes, we use two examples: a relativistic mean field model labelled NL3 [16] and a non-relativistic potential model labelled HLPS, corresponding to “EoS1” in Ref. [2]. Some of their properties are summarized in Table I, where L is related to the derivative of the symmetry energy S_2 with respect to density at the nuclear saturation density, $L = 3n_0(\partial S_2/\partial n)|_{n_0}$. We see that NL3 is an example of a stiff EoS, with a high value of L , producing stars with a high maximum mass and large radius. HLPS is a softer equation of state at $n \lesssim 4n_0$. There is some evidence favoring a soft EoS for nuclear matter: in Ref. [17], values in the range $L = 40$ to 60 MeV were favored from an analysis of constraints imposed by available laboratory and neutron star data. Using data from X-ray bursts, Ref. [18] finds the surface to volume symmetry energy ratio $S_s/S_v \approx 1.5 \pm 0.3$ (See after their Eq. (43) and Table 4), which corresponds to L in the range 22 ± 4 MeV (using Eq. (7) in Ref. [17]).

- **Nuclear/Quark transition.** The nuclear matter to quark matter transition occurs at pressure p_{crit} . We will usually specify its position in terms of the baryon density n_{crit} of nuclear matter at the transition, or the energy density $\varepsilon_{\text{crit}}$ of nuclear matter at the transition. For a given nuclear matter EoS, one can use any of these quantities to specify the location of the transition. We will typically give n_{crit} in units of $n_0 \equiv 0.16\text{fm}^{-3}$.

- **Organization of paper.** In Sec. II we discuss the criteria for stable hybrid stars to exist, as a function of the nuclear matter equation of state and the parameters of our generic quark matter equation of state. We present results for a hard and a soft nuclear matter EoS. In Sec. III we discuss the maximum mass that such hybrid stars can achieve. Sec. IV gives a summary and conclusions.

II. CRITERION FOR STABLE HYBRID STARS

A. Connected hybrid branch

A compact star will be stable as long as the mass M of the star is an increasing function of the central pressure p_{cent} [19]. There will therefore be a stable hybrid star branch in the $M(R)$ relation, connected to the neutron star branch, if the mass of the star continues to increase with p_{cent} when the quark matter core first appears, at $p_{\text{cent}} = p_{\text{crit}}$:

$$\left. \frac{dM}{dp_{\text{cent}}} \right|_{p_{\text{crit}}^+} > 0. \quad (2)$$

This derivative is discontinuous at p_{crit} ; we evaluate it for central pressures infinitesimally above the critical pressure. In this region, the contribution of the quark matter core is dominated by the energy density discontinuity $\Delta\varepsilon$, since the quark core is not large enough for the slope (c_{QM}^2) of $\varepsilon(p)$ to have much influence on the mass and radius of the hybrid star.

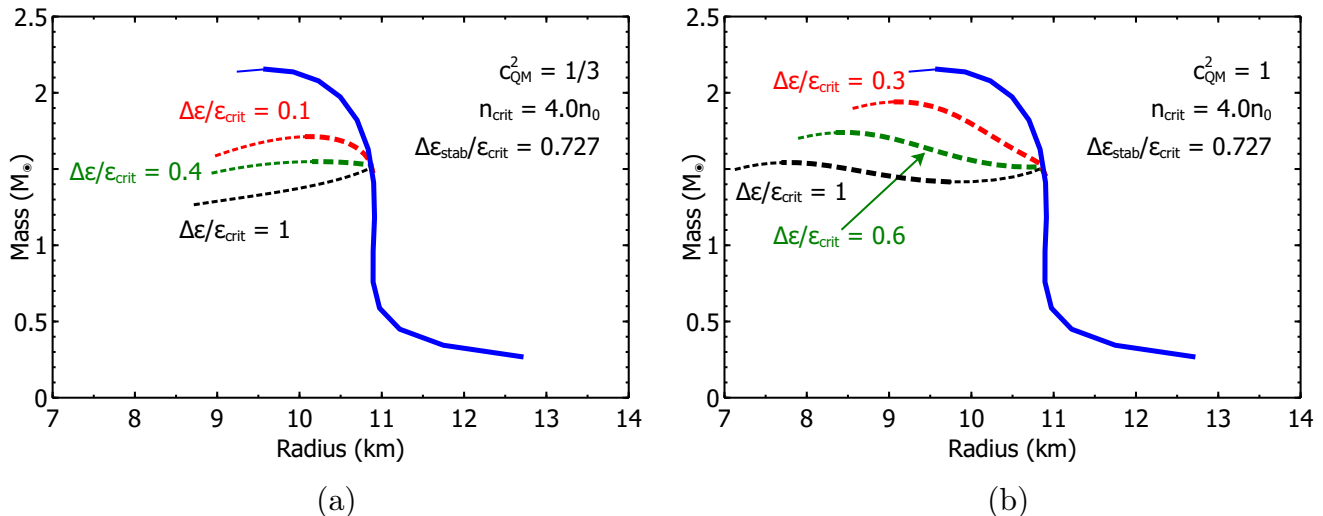


FIG. 2: Mass-radius relations for hybrid stars made of HLPS nuclear matter [2] with the transition to quark matter at $n_{\text{crit}} = 4n_0$ ($n_0 \equiv 0.16 \text{ fm}^{-3}$ is nuclear saturation density) and various values of the energy density discontinuity $\Delta\epsilon$. Thick dashed lines are for stable hybrid stars, thin dashed lines show unstable branches. Case (a): $c_{\text{QM}}^2 = 1/3$; the stable hybrid branch disappears as $\Delta\epsilon$ rises beyond $\Delta\epsilon_{\text{stab}}$. Case (b): $c_{\text{QM}}^2 = 1$; a disconnected stable hybrid branch is present for $\Delta\epsilon \gtrsim \Delta\epsilon_{\text{stab}}$.

If $\Delta\epsilon$ is small then quark matter has a similar energy density to that of nuclear matter, and we expect hybrid stars to be stable when the quark matter core first appears. If $\Delta\epsilon$ is too large, then the star becomes unstable as soon as the quark matter core appears, because the pressure of the quark matter is unable to counteract the additional downward force from the gravitational attraction that the additional energy in the core exerts on the rest of the star. We therefore expect that a stable hybrid star branch will exist if $\Delta\epsilon$ is less than a threshold value $\Delta\epsilon_{\text{stab}}$;

$$\begin{array}{l} \text{QM EoS that gives} \\ \text{connected hybrid branch} \end{array} : \quad \Delta\epsilon < \Delta\epsilon_{\text{stab}} . \quad (3)$$

The situation is illustrated in terms of mass-radius plots in Fig. 2(a), which contains $M(R)$ curves for the HLPS nuclear matter EoS [2] with a transition to quark matter at $n_{\text{crit}} = 4n_0$. In this case, we assume quark matter with $c_{\text{QM}}^2 = 1/3$ and we vary $\Delta\epsilon$. The hybrid branch becomes unstable (mass decreasing with central pressure) when $\Delta\epsilon$ is greater than a critical value $\Delta\epsilon_{\text{stab}}$ which in this case is $0.727\epsilon_{\text{crit}}$.

B. Disconnected hybrid branch

In Fig. 2(b), we illustrate the possibility of a second, disconnected, branch of stable hybrid stars at $\Delta\epsilon > \Delta\epsilon_{\text{stab}}$. The only difference from Fig. 2(a) is that we have increased the speed of sound in quark matter to $c_{\text{QM}}^2 = 1$. The critical value of $\Delta\epsilon$ for a connected hybrid branch is of course unchanged, but if $\Delta\epsilon$ is above that value then

the mass will start rising again when the central pressure rises sufficiently far above p_{crit} , producing a disconnected branch of stable hybrid stars. This constitutes a “third family” [20, 21] of compact stars besides neutron stars and white dwarfs. Such a feature has been noticed in $M(R)$ calculations for specific quark matter models, for example kaon condensed stars [22], quark matter cores from perturbative QCD [23], and color superconducting quark matter cores [24]. Also Ref. [25] demonstrates the transition from connected branch to disconnected branch via twin star collapse. Here we will study the generic features of a quark matter EoS that give rise to this phenomenon.

C. Calculating the critical energy density discontinuity $\Delta\epsilon_{\text{stab}}$

Applying the criterion in Eq. (3) for arbitrary transition pressures, we obtain a function $\Delta\epsilon_{\text{stab}}(n_{\text{crit}})$ which depends only on the nuclear matter equation of state. To evaluate it for a given nuclear matter EoS, we choose a value of n_{crit} and scan through values of the energy density discontinuity $\Delta\epsilon$ to find the largest value that gives a stable hybrid star when the central pressure is pushed just above p_{crit} . We then vary n_{crit} to obtain $\Delta\epsilon_{\text{stab}}(n_{\text{crit}})$.

In Fig. 3 we show the results of a scan through values of $\Delta\epsilon$ for $n_{\text{crit}} = 1.5 n_0$. In this calculation we fix $c_{\text{QM}}^2 = 1/3$, but recall that the value of this parameter does not affect $\Delta\epsilon_{\text{stab}}$. In the example displayed in Fig. 3, the mass M of the star increases with central energy density for small $\Delta\epsilon$, but as $\Delta\epsilon$ increases the rate of increase

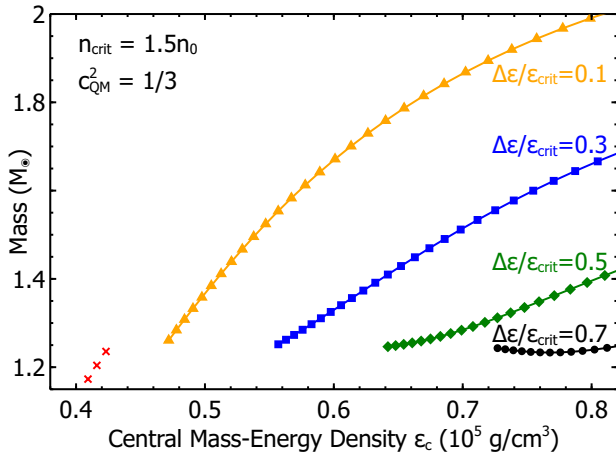


FIG. 3: Calculation of $\Delta\epsilon_{\text{stab}}$: Mass versus the central energy density for quark matter equations of state with various energy density discontinuities $\Delta\epsilon$. The nuclear matter EoS is NL3 and the transition density is fixed at $1.5n_0$. The hybrid star branch becomes unstable when $\Delta\epsilon$ is greater than $\Delta\epsilon_{\text{stab}} \approx 0.6\epsilon_{\text{crit}}$, because the mass then decreases with central pressure when the quark matter core first appears.

of M with $\epsilon_{\text{central}}$ becomes smaller, until $\Delta\epsilon$ reaches the value $\Delta\epsilon_{\text{stab}} \approx 0.6\epsilon_{\text{crit}}$ at which point the mass becomes a decreasing function of central energy density when the quark matter core appears, indicating an unstable star. There is then no stable hybrid star branch connected to the nuclear star $M(R)$ curve. However, as seen in the lowest line in Fig. 3 (for $\Delta\epsilon = 0.7\epsilon_{\text{crit}}$), even if the mass at first decreases with central pressure, it may then turn around and start rising again, indicating the presence of a stable hybrid branch disconnected from the nuclear matter branch. This is the scenario illustrated in Fig. 2(b).

D. “Phase diagram” for hybrid stars

In Fig. 4 we plot a “phase diagram” for hybrid stars, where the control parameters are the baryon density n_{crit} of nuclear matter at the transition to quark matter and the discontinuity $\Delta\epsilon$ in the energy density at the transition. This diagram is obtained by repeating the calculation of Fig. 3 for many values of n_{crit} . We present results for a softer nuclear matter EoS (HLPS) and a stiffer one (NL3).

The region below the solid curve ($\Delta\epsilon < \Delta\epsilon_{\text{stab}}(n_{\text{crit}})$) is where there is a stable hybrid star branch connected to the hadronic star branch. We see that, apart from critical densities close to the upper limit n_{max} where the hadronic star itself becomes unstable, $\Delta\epsilon_{\text{stab}}$ is always about 60% to 80% of the energy density ϵ_{crit} of the nuclear matter at the hadronic-quark transition. (For HLPS we also mark the density $n = 5.458n_0$ at which HLPS becomes unphysical because its speed of sound goes above 1.) Naturally, as n_{crit} comes close to the value n_{max} at which

the hadronic star itself is unstable, the window for a connected branch of stable hybrid stars becomes small and at $n_{\text{crit}} = n_{\text{max}}$ it vanishes.

Above the solid line in Fig. 4 is the region $\Delta\epsilon > \Delta\epsilon_{\text{stab}}(n_{\text{crit}})$ where there is no connected hybrid star branch. The almost-vertical dashed lines in that region demarcate, for two different values of c_{QM}^2 , the regions where there is a separate hybrid star branch, disconnected from the hadronic branch (see Fig. 2b). The properties of such stars depend on the behavior of the quark matter EoS above the deconfining transition, so we show results for two values of c_{QM}^2 .

We see in Fig. 4 that the disconnected branch occurs more readily when the transition density is low, and the speed of sound in quark matter is high. It is also slightly favored by a lower value of $\Delta\epsilon$. Such features were noticed in the context of stars with mixed phases in Ref. [10], which pointed out that the disconnected branch can be understood as follows. When the quark matter core first appears, its greater density creates a strong gravitational pull on the nuclear mantle, which the pressure of the core cannot support, so the star is unstable. However, if the pressure of the core rises fast enough with energy density, a larger core with a slightly higher central density may be able to sustain the weight of the mantle above it. This explains why disconnected branches occur more readily when the transition density is low: the nuclear mantle is then less dense, and easier for the quark core to support. It also explains why the vertical line separating the “disconnected hybrid branch” region from the “no hybrid branch” region moves to the right as c_{QM}^2 increases, since $c^2 = dp/d\epsilon$ is a measure of stiffness: if c_{QM}^2 is larger then the pressure of the core grows more quickly as its density grows, making a large core better able to support the nuclear mantle. Finally, it explains why the line has a slight negative slope: larger $\Delta\epsilon$ makes the quark core heavier, increasing its pull on the nuclear mantle, and making the hybrid star more unstable against collapse.

Ref. [10], assuming a mixed phase, conjectured that the third family (disconnected branch) exists when the speed of sound in quark matter is less than that in the mixed phase. In the case we study, with no mixed phase, the relevant quantity would be the difference between the speed of sound in quark matter c_{QM} and the speed of sound c_{NM} in nuclear matter at the phase transition. However, by comparing Fig. 4 with Fig. 5 we see that, although c_{NM} rises with n_{crit} , it is not the case that the disconnected hybrid branch disappears exactly when $c_{\text{NM}} > c_{\text{QM}}$. For example, for NL3 nuclear matter the speed of sound never reaches 1, but when NL3 nuclear matter is combined with quark matter with $c_{\text{QM}} = 1$ the disconnected hybrid branch still disappears at a finite transition density.

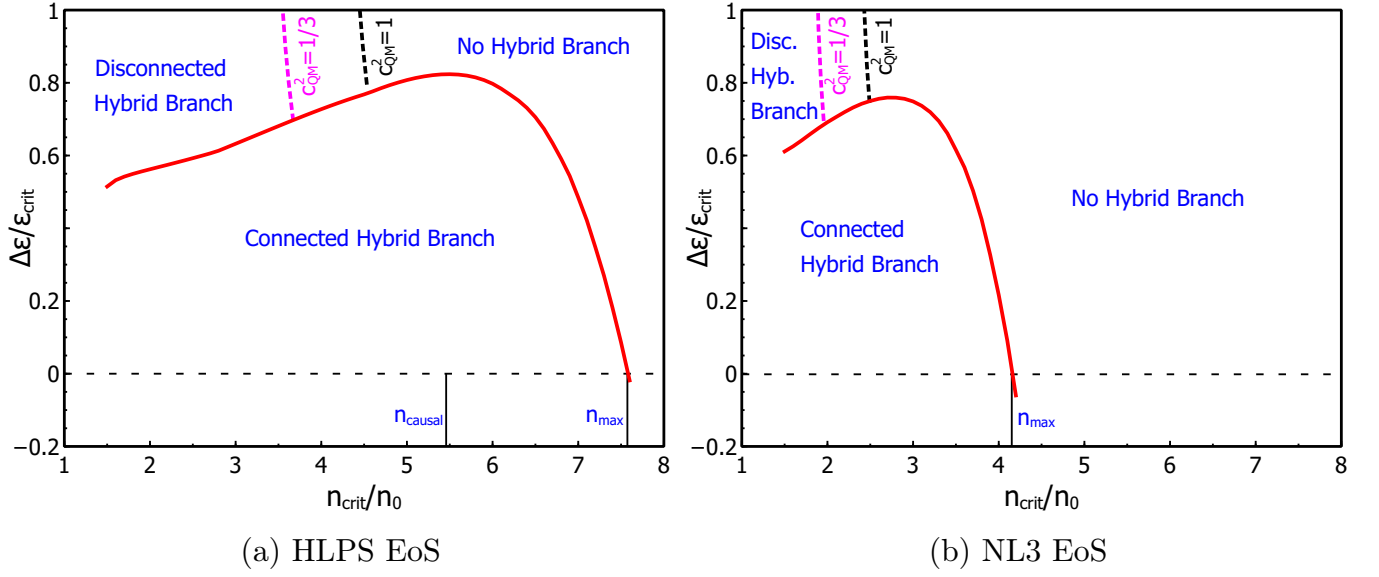


FIG. 4: “Phase diagram” for hybrid star branches in the mass-radius relation. The control parameters are the nuclear matter density n_{crit} at the transition to quark matter, and the energy density discontinuity $\Delta\epsilon$ at the transition. When $\Delta\epsilon < \Delta\epsilon_{\text{stab}}$ there is a connected hybrid branch. This is independent of the speed of sound c_{QM} in the quark matter. When $\Delta\epsilon > \Delta\epsilon_{\text{stab}}$ there may be a disconnected hybrid branch if the critical density is below a threshold value which depends on c_{QM}^2 . We show results for a softer nuclear matter EoS (HLPS [2]) and a harder one (NL3 [16]).

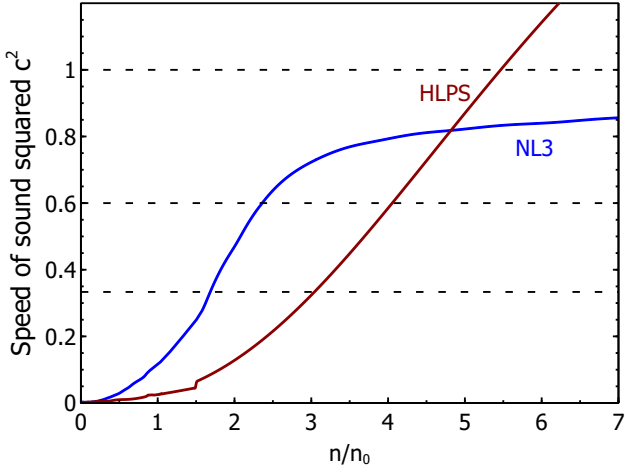


FIG. 5: Speed of sound squared c^2 in the HLPS and NL3 nuclear matter equations of state. At $n \lesssim 4n_0$ HLPS is a softer EoS with a lower speed of sound. At higher density it becomes harder and then acausal.

III. MAXIMUM MASS OF HYBRID STARS

A. Maximum mass and central energy density

The generic quark matter EoS (1) involves three parameters, p_{crit} (or equivalently n_{crit}), $\Delta\epsilon$, and c_{QM} . For each value of these parameters that gives stable hybrid stars, we calculated the maximum mass M_{max} of the hy-

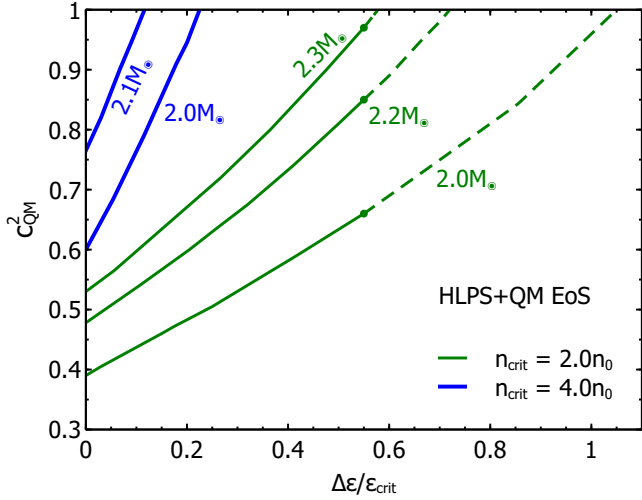
brid star. The results of Ref. [26, 27] lead us to expect that, for a given nuclear matter EoS, the maximum mass is mostly determined by the central energy density of the heaviest star and the speed of sound in the central regions of that star. Specifically,

$$M_{\text{max}} \stackrel{?}{=} y(c_{\text{cent}}) \epsilon_{\text{cent}}^{-1/2}. \quad (4)$$

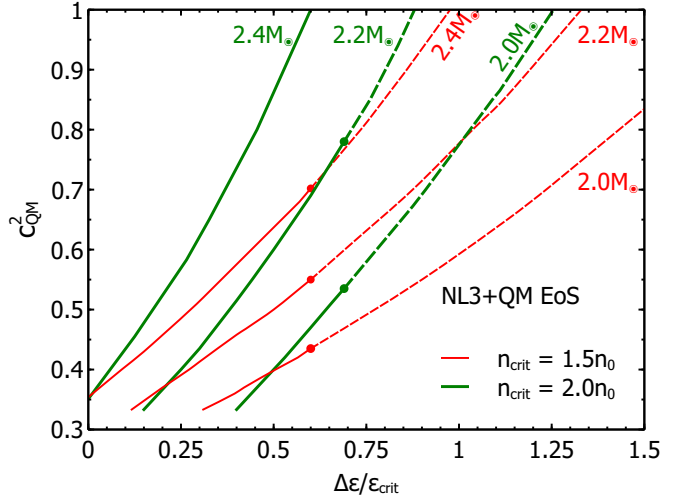
The function $y(c_{\text{cent}})$ can be obtained from Ref. [26] (their Eq. (24) and the associated table). To test Eq. (4), we follow Ref. [26, 27] and plot the maximum mass M_{max} that a stable star can have, as a function of the central energy density in that star ϵ_{cent} . We use the NL3 nuclear matter EoS and repeat the calculation for a range of quark matter equations of state, varying the critical density n_{crit} for the transition to quark matter, the quark matter speed of sound c_{QM} , and the energy density discontinuity $\Delta\epsilon$ at the transition. The range of allowed values of $\Delta\epsilon$, giving stable hybrid stars, is from zero to $\Delta\epsilon_{\text{stab}}(n_{\text{crit}})$.

The results are shown in Fig. 7. There are solid curves showing $M_{\text{max}}(\epsilon_{\text{cent}})$ according to Eq. (4) for $c_{\text{cent}}^2 = 1$, 0.6, and 1/3. Nuclear matter equations of state at high density have c^2 close to 1, hence the maximum masses for pure nuclear matter stars (we show the result for APR [1] nuclear matter as well as NL3 and HLPS) lie close to the $c_{\text{cent}}^2 = 1$ line.

The triangles, squares, and circles show the masses of hybrid stars for a quark matter EoS with relatively low transition densities: $n_{\text{crit}} = 1.5n_0$ (open symbols) or $n_{\text{crit}} = 2n_0$ (solid symbols), and for each of these we vary c_{QM} and $\Delta\epsilon$. The low transition density means



(a) HLPS + QM



(b) NL3 + QM

FIG. 6: Contour plot of the mass of the heaviest hybrid star as a function of quark matter EoS parameters n_{crit} , $\Delta\epsilon$, c_{QM}^2 , for HLPS (left panel) and NL3 (right panel) nuclear matter. The thin (red), medium (green) and thick (blue) lines are for a nuclear to quark transition at $n_{\text{crit}} = 1.5n_0$, $2n_0$, and $4n_0$, respectively. The dashed lines are for stars on a disconnected branch.

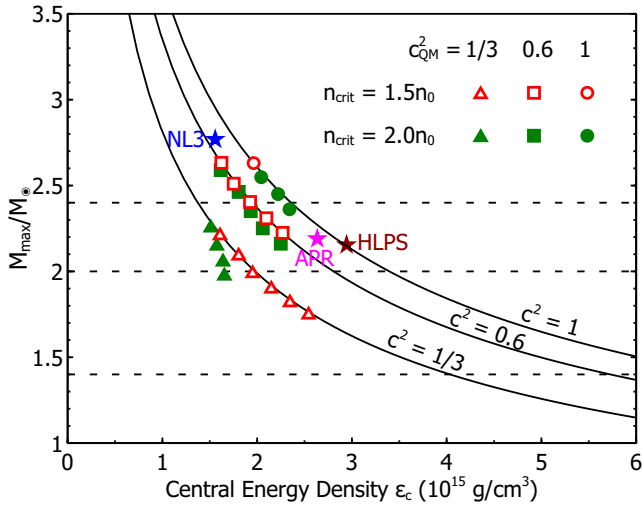


FIG. 7: Mass of the heaviest hybrid star as a function of its central energy density, for various quark matter equations of state (1). The curves are predictions of Ref. [26] for stars whose core-region speed of sound squared is 1, 0.6, and $1/3$. Pure nuclear matter stars for the NL3 and APR equations of state are also plotted.

that these hybrid stars have large quark matter cores, so the speed of sound in the central part of the star is c_{QM} , so the maximum masses should fall on the lines given by Eq. (4) with $c_{\text{cent}} = c_{\text{QM}}$. We see that on the whole this is the case. However, for $n_{\text{crit}} = 2.0n_0$ and $c_{\text{QM}}^2 = 1/3$ (solid triangles) the points fall slightly below the predicted $c_{\text{QM}}^2 = 1/3$ line. This implies that in these stars the effective speed of sound in the core is even

lower than $1/\sqrt{3}$. However, the speed of sound in the nuclear matter mantle at $n \sim 2n_0$ is greater than $1/\sqrt{3}$ (see Fig. 5). It seems reasonable to argue that the first-order phase transition, at which $dp/d\epsilon = c^2$ vanishes, acts like a “soft” region where c is small [12], and drags down the average value of c_{cent} . All hybrid stars have such a transition region, but in these stars the quark matter cores are smaller than in the other cases, so the nuclear-quark matter boundary is closer to the center of the star, and we conjecture that it plays a more important role in determining the maximum mass.

B. How heavy can a hybrid star be?

Using the very generic parameterization (1) of the quark matter EoS, it is possible to get hybrid stars that are heavy enough to be consistent with the recent measurement of a star of mass $2M_\odot$ [28]. In Fig. 6 we show contour plots for the maximum masses of hybrid stars as a function of the parameters of the quark matter EoS, for both the HLPS and NL3 nuclear equations of state. The solid lines are contours of given maximum mass for hybrid stars on a connected branch. These naturally end at $\Delta\epsilon = \Delta\epsilon_{\text{stab}}$ (from Fig. 4). The dashed lines are for stars on a disconnected branch.

In both cases we see that, as expected from Fig. 4, hybrid stars will be heavier if the energy density discontinuity $\Delta\epsilon$ is smaller (so the gravitational pull of the core does not destabilize the star) and the speed of sound in quark matter is higher (so the core is stiffer and can support a heavy star).

For the softer HLPS EoS, whose pure nuclear matter star has a maximum mass of $2.15M_\odot$ (Table I), hybrid

stars can be heavier than the pure nuclear star. This occurs when the transition density is low, and the quark core is a large fraction of the star. In the $M(R)$ relation, the hybrid branch can have a positive dM/dR , so its $M(R)$ curve looks similar to that of a pure quark matter star, rising to a maximum mass star which is heavier and larger than the heaviest pure HLPS star. For the harder NL3 EoS, whose pure nuclear matter star has a maximum mass of $2.77 M_\odot$, hybrid stars are always lighter than the heaviest nuclear matter star.

C. The quark matter mass fraction

For hybrid stars, it is natural to ask is how much of the mass of the star consists of quark matter. We define the quark matter mass fraction $f_q \equiv M_{\text{core}}/M_{\text{star}}$. In Fig. 6(a), along the thick (blue, $n_{\text{crit}} = 4n_0$) contours of constant M_{star} , f_q varies from about 60% at low c_{QM}^2 and $\Delta\epsilon$ to about 70% at high c_{QM}^2 and $\Delta\epsilon$. If the transition pressure is lower then we expect the quark fraction to be larger: along the medium-thickness (green) mass contours for $n_{\text{crit}} = 2n_0$, f_q varies from about 90% at low c_{QM}^2 and $\Delta\epsilon$ to about 96% at high c_{QM}^2 and $\Delta\epsilon$.

In Fig. 6(b), along the medium-thickness (green) mass contours for $n_{\text{crit}} = 2n_0$, f_q varies from about 50% at low c_{QM}^2 and $\Delta\epsilon$ to about 80% at high c_{QM}^2 and $\Delta\epsilon$. Again, if the transition pressure is lower then the quark fraction is larger: along the thin (red) mass contours for $n_{\text{crit}} = 1.5n_0$, f_q varies from about 80% at low c_{QM}^2 and $\Delta\epsilon$ to over 90% at high c_{QM}^2 and $\Delta\epsilon$.

IV. CONCLUSIONS

We studied hybrid stars where there is a sharp interface between two phases with different equations of state. We called the two phases “nuclear matter” and “quark matter”, but our conclusions are valid for any first-order phase transition between two phases with different energy densities.

We found that in this situation there is a simple criterion for determining whether there exists a stable connected branch of hybrid stars. For a given nuclear matter EoS, the only relevant features of the quark matter EoS are the transition density/pressure and the discontinuity in energy density $\Delta\epsilon$ at the transition. A stable connected branch of hybrid stars exists if $\Delta\epsilon$ is less than a critical value $\Delta\epsilon_{\text{stab}}$ which is a function of the nuclear matter EoS and the density at which the transition occurs, and can be easily calculated for any nuclear EoS. Thus for each nuclear matter EoS there is a “phase diagram” (e.g. Fig. 4) in which the line $\Delta\epsilon_{\text{stab}}(n_{\text{crit}})$ delineates the region where there is a connected hybrid branch. In the examples we studied $\Delta\epsilon_{\text{stab}}$ is typically around 60% to 80% of the nuclear matter energy density at the transition, with a rapid drop to zero when the crit-

ical density becomes close to the maximal central density of a pure nuclear matter star.

To study properties of hybrid stars beyond the central pressure where they first appear, we used a fairly generic classical-ideal-gas-type parameterization of the quark matter EoS at densities beyond the transition, and came to the following conclusions.

(1) Even if there is no connected hybrid star branch, there may be a disconnected one if the transition density is low enough so that the nuclear mantle can be supported by a large enough quark matter core (see Fig. 4).

(2) We confirmed that the relationship (4) between the maximum mass of a star and its central energy density, which was proposed in Ref. [26], holds for hybrid stars (see Fig. 7).

(3) We found that it is reasonably easy to get heavy hybrid stars (more than $2 M_\odot$) for reasonable parameters of the quark matter EoS. It requires a not-too-high transition density ($n_{\text{crit}} \sim 2n_0$), low enough energy density discontinuity $\Delta\epsilon \lesssim 0.5\epsilon_{\text{crit}}$, and high enough speed of sound $c_{\text{QM}}^2 \gtrsim 0.4$. A value of c_{QM}^2 that is well above 1/3 is an indication that the quark matter is strongly coupled. For a hard nuclear matter EoS such as NL3 it was somewhat easier to make heavy hybrid stars. For details see Fig. 6. This confirms what was seen in previous calculations for specific models of quark matter (e.g., [11, 13, 15]): in the absence of theoretical or experimental constraints on the quark matter EoS, one can fairly easily vary the unknown parameters of that EoS to obtain heavy hybrid stars. For now, it seems clear that without theoretical advances that constrain the form of the quark matter EoS, measurements of gross features of the $M(R)$ curve such as the maximum mass will not be able to rule out the presence of quark matter in neutron stars.

The generic parameterization of the quark matter EoS that we use in this work provides a general framework for comparison and empirical testing of models of the quark matter EoS. Any particular model can be characterized, at least at densities close to the transition, in terms of the parameters n_{crit} , $\Delta\epsilon$, and c_{QM}^2 , and its predictions for connected hybrid stars will follow from its position in the phase diagram (Fig. 4). If the form of the nuclear matter EoS were established then measurements of the $M(R)$ relation of neutron stars could be directly expressed as constraints on the values of our quark matter EoS parameters.

Our discussion of stars with central pressure well above the transition density relied on the assumption of a density-independent speed of sound, which is a useful starting point for general comparisons of quark matter models, as well as providing specific examples of quark matter equations of state that can yield heavy hybrid stars (Fig. 6). If observations of $M(R)$ for heavy stars turned out to be inconsistent with our parameterization, or if theorists were able to show that the speed of sound in quark matter has significant density depen-

dence, then our approach could be further generalized (at the penalty of introducing more parameters) to allow for that, and to allow for mixed or percolated phases as noted in Refs. [10, 11].

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Appendix A: Ideal gas equation of state

Here we briefly recapitulate (see, e.g., Ref. [12]) the construction of a thermodynamically consistent equation of state of the form in Eq. (1)

$$\varepsilon(p) = \varepsilon_0 + \frac{1}{c^2}p. \quad (\text{A1})$$

We start by writing the pressure in terms of the chemical potential

$$\begin{aligned} p(\mu_B) &= A\mu_B^{1+\beta} - B, \\ \mu_B(p) &= \left(\frac{p+B}{A} \right)^{1/(1+\beta)}. \end{aligned} \quad (\text{A2})$$

Note that we have introduced an additional parameter A with mass dimension $3 - \beta$. The value of A can be varied without affecting the energy-pressure relation (A1). When constructing a first-order transition from some low-pressure EoS to a high-pressure EoS of the form (A1), we must choose A so that the pressure is a monotonically increasing function of μ_B (i.e. so that the jump in n_B at the transition is not negative). The derivative with respect to μ_B yields

$$n_B(\mu_B) = (1 + \beta) A \mu_B^\beta \quad (\text{A3})$$

and using $p = \mu_B n_B - \varepsilon$, we obtain the energy density

$$\varepsilon(\mu_B) = B + \beta A \mu_B^{1+\beta}. \quad (\text{A4})$$

Then Eq. (A2) gives energy density as a function of pressure

$$\varepsilon(p) = (1 + \beta)B + \beta p \quad (\text{A5})$$

which is equivalent to Eq. (A1) with $1/c^2 = \beta$ and $\varepsilon_0 = (1 + \beta)B$.

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